JEE-Main-31-01-2024 (Memory Based)
[MORNING SHIFT]

Mathematics

Question: Solve differential equation:

$$\frac{dx}{dy} = x\left(\ln x - \ln y + 1\right)$$

Answer:

Solution:

$$\frac{dx}{dy} = \frac{x}{y}\left(\ln\frac{x}{y} + 1\right)$$

Put $x = vy$

$$\Rightarrow v + y\frac{dv}{dy} = v\left(\ln v + 1\right)$$

$$\Rightarrow \int\frac{dv}{v\ln v} = \int\frac{dy}{y}$$

$$\Rightarrow \ln\left|\ln v\right| = \ln\left|y\right| + \ln c$$

$$\Rightarrow \left|\ln\frac{x}{y}\right| = c\left|y\right|$$

Question: $\lim\limits_{x\to 0}\frac{e^{2|\sin x|} - 2|\sin x| - 1}{x^2} = ?$

Options:

(a) Does not exist

(b) 2

(c) 1

(d) -1

Answer: (b)

Solution:

$$\lim_{x\to 0}\frac{e^{2|\sin x|} - 2|\sin x| - 1}{x^2}$$

$$= \lim_{x\to 0}\frac{e^{2|\sin x|} - 2|\sin x| - 1}{|\sin x|^2} \times \frac{\sin^2 x}{x^2}$$

$$= \lim_{t\to 0}\frac{e^{2t} - 2t - 1}{t^2} \qquad t = |\sin x|$$

$$= \lim_{t\to 0}\frac{2.e^{t} - 2}{2t} = \lim_{t\to 0}\frac{2\left(e^{2t} - 1\right)}{2t} = 2$$

$192\sin^2 a$.

Answer: 48.00

Solution:

$|\bar{a}|=1, |\bar{b}|=4,\ \bar{a}.\bar{b}=2 \rightarrow \cos\theta=\frac{1}{2} \rightarrow \theta=\frac{\pi}{3}$

$\bar{c}=2\bar{a}\times\bar{b}-3\bar{b}$

$|\bar{c}|^2=4|\bar{a}\times\bar{b}|^2+9|\bar{b}|^2$

$|\bar{c}|^2=4|\bar{a}\times\bar{b}|^2+9|\bar{b}|^2$

$=4|\bar{a}|^2|\bar{b}|^2\sin^2\theta+9|\bar{b}|^2$

$=4\times1\times16\times\frac{3}{4}+9\times16$

$=48+122=192$

$\bar{b}\cdot\bar{c}=0-3|\bar{b}|^2$

$4\times8\sqrt{3}\cos\alpha=-3\times16$

$\cos\alpha=\frac{-\sqrt{3}}{2}$

$192\sin^2\alpha=48$

Question: Let ‘S’ be the set of positive integral values of a for which $\frac{ax^2+2(a+1)x+9a+4}{x^2+8x+32}<0,\ \forall x\in R.$ Then, the number of elements in ‘S’ is

Answer:

Solution:

$\frac{a^2+2(a+1)x+4a}{x^2+8x+32}<0\forall x$

$D^r>0\ \forall x$

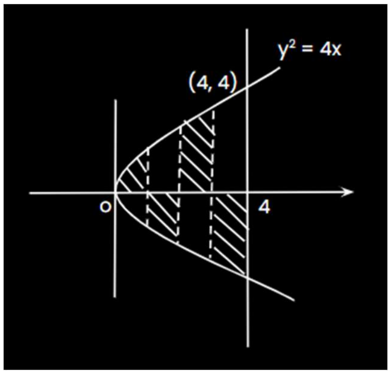
N^rcan be -ve for all x only if

a is -ve so no +ve integral vaue of a.

Question: There was on area $\left\{y^2<4x, x<4, \frac{xy(x-1)(x-2)}{(x-3)(x-4)}<0\right\}$.

Answer: $\frac{32}{3}$

Solution:



$y > 0$

$x(x-1)(x-2)(x-3)(x-4) > 0$

$y < 0$

$x(x-1)(x-2)(x-3)(x-4) < 0$

$\text{Area} = \frac{2}{3} \times 16 = \frac{32}{3}$

Question: If $f(x) = \begin{vmatrix} x^3 & 2x^2+1 & 1+3x \\ 3x^2+2 & 2x & x^3+6 \\ x^3-x & 4 & x^2-2 \end{vmatrix}$ for all x ∈ R, then 2f(0) + f'(0) is equal to

Answer: 42.00

Solution:

$$f(0) = \begin{vmatrix} 0 & 1 & 1 \\ 2 & 0 & 6 \\ 0 & 4 & -2 \end{vmatrix} = 8 + 4 = 12$$

$$f'(0) = \begin{vmatrix} 0 & 0 & 3 \\ 2 & 0 & 6 \\ 0 & 4 & -2 \end{vmatrix} + \begin{vmatrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 4 & -2 \end{vmatrix} + \begin{vmatrix} 0 & 1 & 1 \\ 2 & 0 & 6 \\ -1 & 0 & 0 \end{vmatrix}$$

$= 24 + 0 + (-6) = 18$

$2f(0) + f'(0) = 42$

Question: If $f(x) = \frac{4x+3}{6x-4}, x \neq \frac{2}{3}$. Find $(fof)(x) = g(x)$, where $g : r - \left[\frac{2}{3} \to R \to \left\{\frac{2}{3}\right\}\right]$ then $\left(g\left(g\left(g(4)\right)\right)\right)$ is equal to

Answer: 4.00

Solution:

$f(x) = \frac{4x+3}{6x-4}$

$$g(x)=f(f(x))=\frac{4\left(\frac{4x+3}{6x-4}\right)+3}{6\left(\frac{4x+3}{6x-4}\right)-4}$$

$$=\frac{16x+12+18x-12}{24x+18-24x+16}=\frac{34x}{34}=x$$

$$g(g(g(4)))=4$$

Question: $\sum_{r=1}^{10}\frac{r}{1-3r^2+r^4}=S$. Find S.

Answer: $\frac{-55}{109}$

Solution:

$$\sum_{r=1}^{10}\frac{r}{1-3r^2+r^4}=\sum_{r=1}^{10}\frac{r}{\left(r^2-1\right)^2-r^2}$$

$$=\sum_{r=1}^{10}\frac{r}{\left(r^2+r-1\right)\left(r^2-r-1\right)}$$

$$=\sum\frac{1}{2}\left[\frac{1}{r^2-r-1}-\frac{1}{r^2+r-1}\right]$$

$$=\frac{1}{2}\left[\left(\frac{1}{-1}-\frac{1}{1}\right)+\left(\frac{1}{1}-\frac{1}{5}\right)+\ldots\left(-\frac{1}{109}\right)\right]$$

$$=\frac{1}{2}\left[-1-\frac{1}{109}\right]=\frac{-110}{2\times109}=\frac{-55}{109}$$

Question: If the system of linear equation x - 2y + z = -4; 2x + αy + 3z = 5 and 3x - y + βz = 3 has infinitely many solutions then 12α + 13β is equal

Answer: 58.00

Solution:

$$x-2y+z=4$$

$$2x+\alpha y+3z=5$$

$$3x=y+\beta z=3$$

$$\begin{vmatrix}1 & -2 & 1\\2 & \alpha & 3\\3 & -1 & \beta\end{vmatrix}=\alpha-18-2-3\alpha+3-4\beta=0$$

$$\alpha\beta-3\alpha+4\beta-17=0$$

$$(\alpha+4)(\beta-3)=5$$

Also, $D_2 = \begin{vmatrix} 1 & -4 & 1 \\ 2 & 5 & 3 \\ 3 & 3 & \beta \end{vmatrix} = 0$

$\Rightarrow s\beta - 36 + 6 - 15 - 9 + 8\beta = 0$

$13\beta - 54 = 0$

$D_3 = \begin{vmatrix} 1 & -2 & -4 \\ 2 & \alpha & 5 \\ 3 & -1 & 3 \end{vmatrix} = 0$

$\rightarrow 3\alpha - 30 + 8 + 12\alpha + 5 + 12 = 0$

$15\alpha - 5 = 0 \Rightarrow \alpha = \frac{1}{3}$

$12\alpha + 13\beta = 4 + 54 = 58$

$\Delta = \left(\frac{1}{3} + 4\right)\left(\frac{54}{13} - 3\right)$

$= \frac{13}{3} \times \frac{15}{13} = 5$

Also,

$D_1 = \begin{vmatrix} -4 & -2 & 1 \\ 5 & \alpha & 3 \\ 3 & -1 & \beta \end{vmatrix}$

$= -4\alpha\beta - 18 - 5 - 3\alpha - 12 + 10\beta$

$= -4\alpha\beta - 3\alpha + 10\beta - 35$

$= -\frac{4\beta}{3} - 1 + 10\beta - 35$

$= \frac{26\beta}{3} - 36 = 0$

Question: $f(x) = \begin{cases} g(x), & x \le 0 \\ \left(\frac{x+1}{x+2}\right)^{\frac{1}{x}}, & x > 0 \end{cases}$, where $g(x)$ is linear function and $f(x)$ is continuous at $x = 0$. Also $f'(1) = g(-1), g(0) = f(0)$ then find the value of $g(3)$?

If ans is $\frac{a}{b}\ln\left(\frac{\alpha}{\beta}e^{\frac{1}{3}}\right)$ find $\alpha + \beta + a + b$

Answer: 15.00

Solution:

$$f(x)=\begin{cases} g(x), & x\le 0 \\ \left(\dfrac{x+1}{x+2}\right)^{\frac{1}{x}}, & x>0 \end{cases}$$

$$f\left(0^+\right)=0;\ f\left(0^-\right)=g\left(0^-\right)=0$$

$$\Rightarrow g(x)=mx$$

$$f'(1)=\frac{d}{dx}e^{\frac{1}{x}\left[\ln(x+1)-\ln(x+2)\right]}$$

$$=\frac{2}{3}\times\left[1\cdot\left(\frac{1}{2}-\frac{1}{3}\right)-1\left(\ln\frac{2}{3}\right)\right]$$

$$=\frac{2}{3}\left[\frac{1}{6}-\ln\frac{2}{3}\right]=\frac{1}{9}-\frac{2}{3}\ln\frac{2}{3}$$

$$=g(-1)=-m$$

$$g(3)=3m=2\ln\frac{2}{3}-\frac{1}{3}$$

$$=2\left[\ln\frac{2}{3}-\ln e^{\frac{1}{3}}\right]$$

$$=2\left[\ln\frac{2}{2e^{\frac{1}{3}}}\right]$$

$$=\ln\frac{4}{9e^{\frac{1}{3}}}$$

$$a=b=1,\ \alpha=4,\ \beta=9$$

$$\alpha+\beta+a+b=15$$

Question: 3 rotten apples are mixed with 15 normal apples. Let the random variable be defined as number of rotten apples on picking 3 apples with replacement. Find variance of x

Answer: $\frac{5}{12}$

Solution:

3 rotten apples, 15 good apples

$$p=\frac{1}{6},q=\frac{5}{6}$$

$$\text{Variance }=npq=3\times\frac{1}{6}\times\frac{5}{6}=\frac{5}{12}$$

drawn with replacement. Find the probability such that the first drawn ball is red and second drawn ball is white?

Answer: $\frac{4}{75}$

Solution:

10 R, 30 W, 15 O, 10 or 20 B

P(1st Red 2nd W) $= \frac{10}{65} \times \frac{30}{65} = \frac{12}{169}$

or $= \frac{10}{75} \times \frac{30}{75} = \frac{12}{225} = \frac{4}{75}$

Question: Using the word "DISTRIBUTION " . Find the no of ways of selection of 4 letters

Answer: 191.00

Solution:

D, 3I, 2T, R, B, U, O, N, S

4 different: ${}^{9}C_{4} = 126$

2 alike, 2 different: ${}^{2}C_{1} \times {}^{8}C_{2} = 2 \times 28 = 56$

3 alike, 1 different = $1 \times {}^{8}C_{1} = 8$

2 alike, 2 alike = 1

Total = 191

Question: $\sum_{r=0}^{n} \frac{{}^{n}C_{r} \cdot {}^{n}C_{r}}{r+1} = \alpha$, $\sum_{r=0}^{n} \frac{{}^{n}C_{r} . {}^{n+1}C_{r}}{r+1} = \beta$. If $4\beta = 7\alpha$, then find n.

Options:

(a) 2

(b) 4

(c) 6

(d) 5

Answer: (c)

Solution:

$$\sum_{r=0}^{n} \frac{{}^{n}C_{r}\, {}^{n}C_{r}}{r+1} = \alpha$$

$$\sum_{r=0}^{n} \frac{{}^{n}C_{r}\, {}^{n+1}C_{r}}{r+1} = \beta$$

$$\alpha = \frac{1}{n+1} \sum {}^{n+1}C_{r+1}\, {}^{n}C_{r} = \sum \frac{{}^{2n+1}C_{n}}{n+1}$$

$$\beta = \frac{1}{n+1} \sum {}^{n+1}C_{r+1}\, {}^{n+1}C_{r} = \sum \frac{{}^{2n+2}C_{n}}{n+1}$$

$$\frac{7}{n+1}\left(\sum {}^{2n+2}C_{n}\right) = \frac{4\left(\sum {}^{2n+2}C_{n}\right)}{n+1}$$

$$7\,{}^{2n+1}C_n = \frac{4\times 3n+2}{n}\left(\sum {}^{2n+2}C_{n-1}\right)$$

$$\frac{7}{8}\frac{n}{n+1}\times\frac{{}^{2n+1}C_n}{{}^{2n+1}C_{n-1}} = 1$$

$$\left(\frac{2n+1-(n)+1}{n}\right) = \frac{8(n+1)}{7n}\quad \frac{{}^{n}C_r}{{}^{n}C_{r-1}} = \frac{n-r+1}{r}$$

$$\frac{n+2}{r} = \frac{8(n+1)}{7r}$$

$$7n+14 = 8n+8$$

$$n = 6$$